2024 Spring — Geometric Representation Theory (1)

COURSE SYLLABUS

Logistics

Instructor: Lin Chen (kylinchen@tsinghua.edu.cn, Shuangqing Complex Building 508).
Teaching Assistant: Wenhan Dai (dwh23@mails.tsinghua.edu.cn).

Time & Venue: Mondays 9:50–12:15 am; 6B105.
Course Website: https://windshower.github.io/linchen/teaching/s2024.html.

Office Hours: To be determined after the voting.

WeChat Group: Scan the following QR code (valid until February 29). Note that everything is to be announced on both WebLearning and the WeChat group simultaneously.

Course Description

Referring to [Gai05] and [Hum08], the course is to introduce Kazhdan–Lusztig theory, which is the foundation of the modern geometric representation theory. Hopefully, the course is to cover the BGG category $\mathscr{O}$ of representations of semisimple Lie algebras, the theory of $\mathfrak{D}$-modules, and the conjecture of Kazhdan–Lusztig as well as its proof.

References:

[Gai05] Dennis Gaitsgory, *Course Notes for Geometric Representation Theory*, 2005, available at https://people.mpim-bonn.mpg.de/gaitsgde/267y/cat0.pdf.

[Hum08] James E. Humphreys, *Representations of Semisimple Lie Algebras in the BGG Category $\mathscr{O}$*, in Graduate Studies in Mathematics, 2008, American Mathematical Society.

Gradings

The final grade of each student will be determined based on the following components, with proportions adjustable after score release; that is,

$$\text{Final Grade} = \lambda_{\text{H}} \cdot (\text{H}) + \lambda_{\text{P}} \cdot (\text{P}) + \lambda_{\text{O}} \cdot (\text{O}),$$

where $30\% \leqslant \lambda_{\text{H}} \leqslant 60\%$ and $0 \leqslant \lambda_{\text{P}}, \lambda_{\text{O}} \leqslant 50\%$.

(H) **Homework** (30% – 60%).
   - These assignments are designed to reinforce class concepts and will be given every two weeks throughout the semester, excluding the first and last two weeks.
   - Each assignment consists of 5 problems discussed in class and should be submitted by Monday 23:59 of the even-numbered weeks (first assignment due on **March 18**).

- ○ Collaborative work is allowed but should be clearly acknowledged. Teaching assistants will grade assignments and release solutions on odd-numbered Mondays.
- ○ Late submissions will incur deductions. Assignments submitted after the solution release will not be graded. Exceptions for late submissions due to special circumstances will be considered.
- ○ The lowest score among the 6 assignments will be dropped.

(P) **Presentation** ($\leqslant 50\%$).
- ○ The presentations, scheduled for the final two weeks (May 27 and June 3), offer an opportunity for groups of 1 to 3 students to deliver short talks lasting 15–30 minutes each.
- ○ Topics should relate to geometric representation theory but must extend beyond the course material. Students may choose their topics or seek suggestions during office hours. Some topic recommendations will also be provided during class.
- ○ Approval of topics and groups' participation must be confirmed with the instructor before the Labour Day holiday (May 1).

(O) **Oral Assessment** ($\leqslant 50\%$).
- ○ Details regarding the timing and format of oral assessments will be announced later.
- ○ These assessments will evaluate students' understanding of the course material and may allow the use of reference materials and electronic devices.
- ○ Points will not be deducted for correct responses provided within the allotted time, and adjustments can be made to answers after errors are identified.